



P vs NP... Are they same?

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Abstract

P vs NP is possibly one of the most crucial problems's of our era owing to the fact that it directly affects one of the most basic things of our modern day survival, the Internet security. The proof will be surely a big blow to the RSA ciphering–deciphering technology but it the way it is! Genuine apologies for $P = NP$. As for as mathematical gain is concern it is a result that opens a search for solution of those 300 plus NP complete problems and much more. The present proof resolves $P = NP$ by the solution of NP complete Hamiltonians path problem in polynomial time. The proof is using topology and simple geometry. Hence $P = NP$; solved for the Hamiltonians path problem or Traveling salesman problem as it is called so. NP complete Hamiltonian's path problem has a polynomial time solution, i.e. $P = CN^4$ for HPP.

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Keywords: Polynomial time; Non-deterministic polynomial time; Hamiltonians path problem; Traveling salesman's problem

Main content:

Statement: Mathematically P vs NP states

$$P = NP \quad \text{or} \quad P \neq NP$$

i.e., whether or not P is equal to NP

Meaning of P and NP: P states for polynomial time problems, problems that can be effectively solved in polynomial time by using a computer. Polynomial time means reasonable time. P problems are characterized by a polynomial equation. i.e.

$$P = CN^K$$

Here P = polynomial time, i.e. time required to solve a P type problem, where C is the constant, N is the number of outcomes, K is the order of P type problem.

Hence, P represents a class of polynomial in which total number of outcomes are proportional to a integral power of inputs. NP problems are those in which time required to get a solution is unreasonably large, though the cases are too much, each case itself needs trivial arithmetic only.

Only problem in number of cases is NP literally means non-deterministic polynomial time problem. A computer in polynomial or reasonable time cannot handle NP problem. More often than not there are NP prob-

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Nomenclature

P	polynomial time problem
NP	non-deterministic polynomial time problem
$=$	is equal to
π	Pi (180 in trigonometry)
$\neq$	is not equal to
N^K	' N ' raised to power ' K '
$N!$	N factorial
nC_K	combination of ' n ' things taken ' K ' at a time
I	set of integers.
HPP	Hamiltonians path problem
TSP	Traveling salesman problem

lems that may take centuries for a full solution by Brute-force method, i.e. by the method of checking all options. There are about 300 NP complete problems. A NP complete problem is one that is father of all NP problems. It means that if one NP complete problem is solvable in polynomial time so it can be any other problem.

Mathematically NP completeness is the generalization of NP problems. In order to prove or disprove $P = NP$, we have to prove or disprove it for one of those 300 NP complete general, problems.

Result: $P = NP$; established for Hamiltonian's path problem or Traveling salesman's problem. The Hamiltonian's path problem or Traveling salesman's problem is solvable in polynomial time of forth degree at most, i.e. for HPP or TSP $P = CN^4$ at most.

Proof. Hamiltonian's path problem (HPP) or Traveling salesman problem (TSP) is a known NP complete problem. We would established that it is solvable in polynomial time of forth degree at most. Before we state TSP or HPP. $\square$

TSP: Suppose there is a salesperson that has to visit several cities in order to sell business. He has the specified map of all the cities that come in his way. Obviously his problem is to find shortest possible route that cover all the cities.

A high school problem!

We can name all the routes and get the answer instantaneously. But the bone in the dish is not summing the distances from city to city. It is the number of such cases.

For ' n ' cities total cases turn out to be $n!$

$\Rightarrow$ Even for modest 100-city tour there are 100! cases.

These cases are too large for a computer to handle. It may take decade for a fastest computer on earth to find shortest possible route for say 1000 cities only. Actually computers can handle polynomial time processes, i.e. where $P = C \times N^K$ These polynomials does not grow that fast if ' N ' is the variable, i.e. number of cities $P = C \times N^{10}$ (say). Does not grows as fast as say $P = C \times 3^N$. Here latter are called exponential time processes.

After them comes NP processes. Now, we will prove that HPP or TSP is solvable in polynomial time using geometrical and topological properties of polygons applied on topologically equivalent maps.

Mathematically total cases for TSP are reducible to CN^4 from $N!$ Our solution is geometrical. For a start, we assume that maps available are topologically correct, i.e. in which relative distances matter and no scaling is required.

For e.g. in Fig. 1 $d(a_i a_j) < d(a_{10} a_j) < d(a_4 a_j)$, etc.

Here $d(a_i a_j)$ is usual distance function measuring distance between a_i and a_j relative to distance between a_{10} and a_j , etc. These maps are topological maps only. The issue of shortest route. We will state and prove three results that will be used further in proof. We start with few definitions.

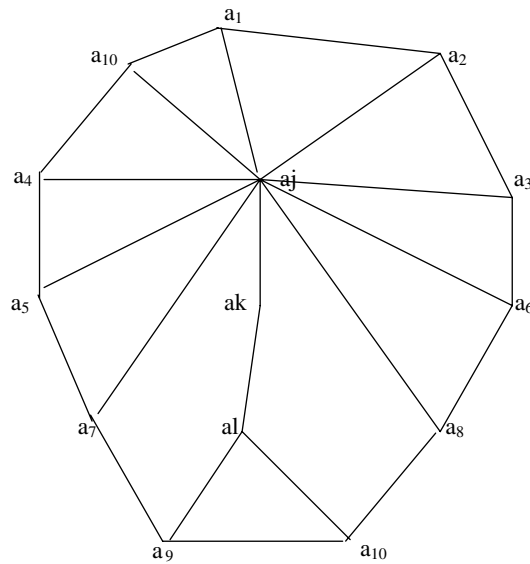


Fig. 1.

68 *Convex polygon:* A convex polygon is one in which all the angles are less than 180° .

69 *Standard convex polygon:* A standard convex polygon or SCP for short is one in which all the internal
 70 angles are between 90° and 180° . A peculiar property of SCP is that all diagonals are greater than the two
 71 sides forming it, or adjacent sides to it. It is easy to establish since in a right triangle hypotenuse is diagonal
 72 or greatest side and as the opposite angle grows the diagonal side dilates. Now, we are in a position to state
 73 our former result.

74 **Theorem 1.** *For all points lying on the periphery of a SCP, the shortest route between them is through the*
 75 *peripheral path.*

76 **Proof.** This can be established without any trouble. Any other route other than peripheral route will include
 77 one or more diagonals. As stated before in SCP the diagonals are larger than the forming sides. Hence, if three
 78 diagonals replace three sides they would increase the net distance. $\square$

79 We can prove it rigorously too as follows:

80 Let the original route value along periphery be ' N '.

81 Now let A_1 is joined to A_7 .

82 A_8 is joined to A_3 and A_4 .

83

84 $\therefore$ New network distance is

85 $N - A_1A_8 - A_8A_7 + A_7A_1 + A_3A_8 + A_4A_8 - A_3A_4$.

86 Now $A_1A_7 > A_1A_8$ (A_1A_7 IS diagonal of SCP). $A_3A_8 > A_1A_8$ (A_3A_8 is the adjacent of SCP).

87 Also $A_8A_4 > (A_8A_4$ is the adjacent diagonal of SCP).

88 $\Rightarrow$ The Net network distance increases.

89

90 Hence for the points lying on a standard polygon the shortest route is along the periphery. Our next the-
 91 orem is too a result about SCP.

92 **Theorem 2.** *A maximum of three points lying on a convex polygon disqualify for being a point on standard*
 93 *polygon.*

94 Proof. We know for any convex polygon sum of all internal angles is $(2n - 4)$ right angles.

97 Since for three vertices we have one triangle.

98 For four vertices we have two triangles.

99 $\vdots$

100 For ' n ' vertices we have ' $n - 2$ ' triangles.

101

102 As proved before the minimum condition for existence of a diagonal route is $\theta > \pi/2$ or 90° , where θ is
103 internal angle.

104 Now for a polygon of ' n ' vertices only three angles can have internal angle less than $\pi/2$ or a right angle.

105 $\Rightarrow$ For example if four internal angles are less than $\pi/2$.

106 $\Rightarrow$ Their sum is less than 2 or 4 right angles.

107 $\Rightarrow$ Sum of other ' $N - 4$ ' vertices is greater than $(2n - 4 - 4)$ right angles.

108 $\Rightarrow$ Sum of ' $N - 4$ ' vertices is $>(2n - 8)$ right angles.

109

110 Or sum of one vertices is >2 right angles which is not allowed, since a vertex has a triangle whose sum is
111 180° or two right angles.

112 $\Rightarrow$ Maximum of three points of a convex polygon can deviate from being on a SCP.

113 **Theorem 3.** All points lying on a convex polygon have shortest route through its periphery.

114 We have proved in Theorem 1 the result for SCP. From Theorem 2 maximum of a three points can deviate
115 from being on SCP (see Fig. 2).

116 $\Rightarrow$ From Fig. 3 Let P_1, P_2, P_3 deviate from the rule.

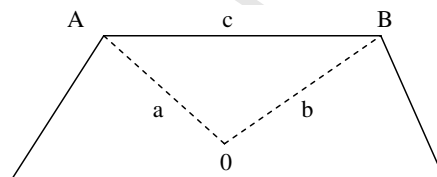


Fig. 2.

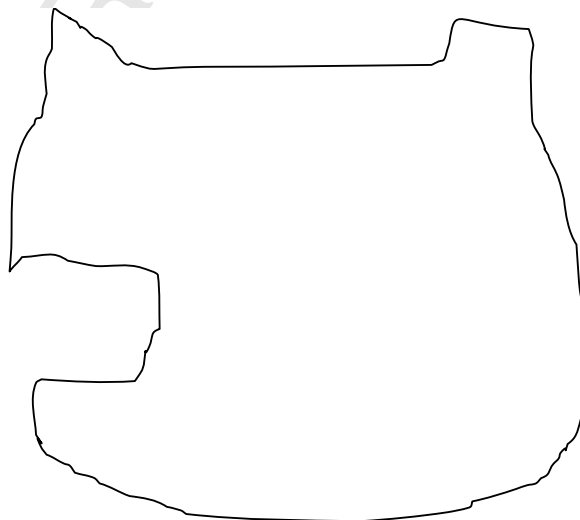


Fig. 3.

We will join these points to the points on the periphery that induce the smallest network distance. Network distance is measured through ' $a + b - c$ ' rule. Here a, b are the distances, which are added to the network and c is the reducing distance.

For e.g. for Pt. ABP_1 , where AP_1 is a – adding distance; BP_1 is b – adding distance; AB is c – subtracting distance. So in any case the point will come along the periphery.

1. The general domain

How can we use the before proved Results 1, 2, and 3 to get the shortest route?

Here is the answer.

Consider the general domain of figure one. Our basic approach for the shortest route is that we start from the shortest and keep it shortest all the while. We start with the outer most mesh of one map and join them so the maximum numbers of destinations lie on a convex polygon. From Theorem 3 the shortest route lies on the periphery for these cities.

Our next object is to join to these branches the points which are nearest to them than any other two points. For this we calculate ' $a + b - c$ ' for all ' n ' cities for all the branches of outer mesh, if ' $a + b - c$ ' is minimum for any of the branches we join it to the branch.

We would again like to define $a + b - c$ rule, where a is the adding distance; b is the adding distance; c is the subtracting distance.

$\therefore a + b - c =$ Net addition to the existing network due to new point ' O '.

As seen above for the section AO , BO is adding distances and AB is subtracting. We repeat the process for new joined branches till we reach a network that looks like.

The above network has following properties.

- (i) It is the shortest route between the points on the network joined so for Fig. 4.
- (ii) All the points that are left are either nearer to them selves or to branches other than on the network. These may be called invisible diagonals. The name pops up as they are hypothetical diagonals which can still be joined between the points on the already existing network of Fig. 5.

2. The next network case

After we have the original network intact we start with other independent points, independent in the sense they are nearer to them selves than to any of the points on the exiting network. We repeat the same process of the general domain till all the points gets exhausted.

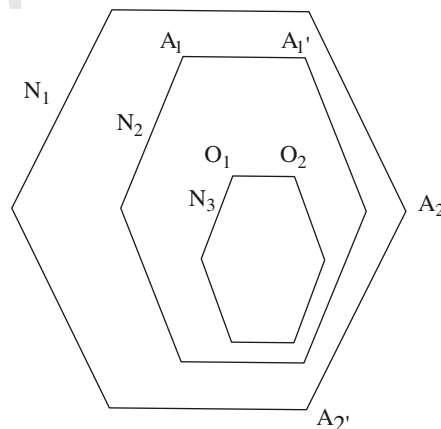


Fig. 4.

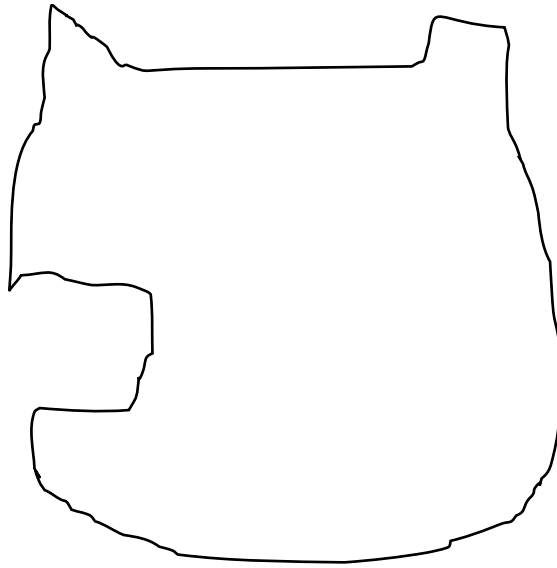


Fig. 5.

147 So our net shortest route may now look like Fig. 6, we have taken three network for simplicity. The three
 148 networks are respectively the shortest route between the particles of the corresponding networks we now use
 149 segment rule to join these networks. It is that the networks are joined via the closest segment. The segment
 150 length is calculated as follows:

151 $a + b - c - d.$

152 Here a, b are adding distance and c, d are subtracting distances. Suppose we have to join $A_1A_1^1$ to O_1O_2 ∴

153 The net adding distance is

154 $a = A_1O_1,$

155 $b = A_1^1O_2$ and net subtracting distance is $A_1A_1^1$ and O_1O_2 .

156

157 For whichever two segments the ' $a + b - c - d$ ' is minimum we join them. Next comes the case of hypo-
 158 thetical diagonals. Now our shortest route may look like Fig. 7 (For simplicity exact geometrical figures

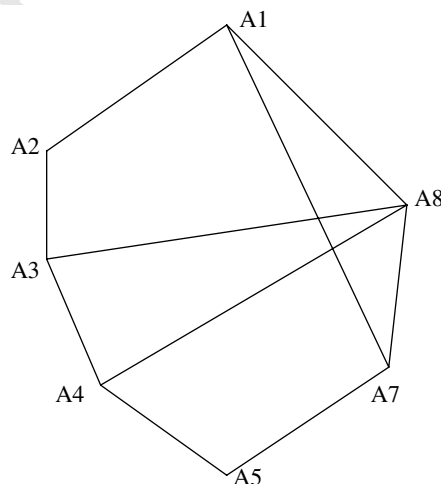


Fig. 6.

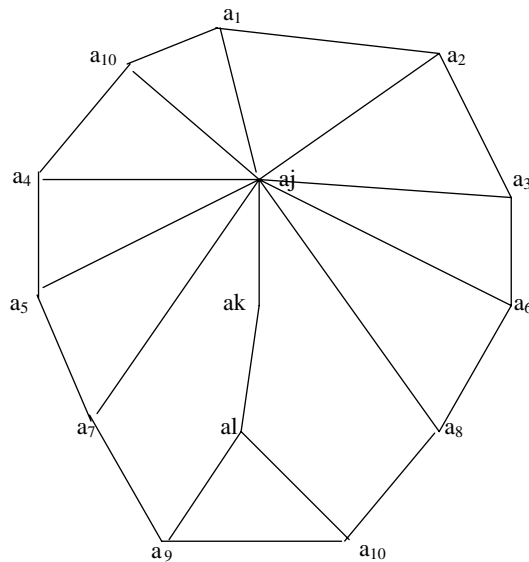


Fig. 7.

are assumed, the original networks are usually distorted enough). The case of hypothetical diagonals will be dealt after the present case of next network.

3. The case of hypothetical diagonals

Again we may have the shortest route between the points one more query arises. We may join any two points and consider a hypothetical diagonal. Then we may join the nearest points to this hypothetical diagonal and calculate the whole mesh if it comes lower than previous we take this hypothetical route as a new shortest route. The process is repeated for all points' s and we arrive at the shortest possible route between the points.

4. The last check

Till now we have got a shortest route between the existing points. But to prove it is the shortest route indeed we check for all points $a + b - c$ again if there is any point not joined to the nearest branch it will come to our notice. So the last check infect establishes that no other subsequent alterations to the position of the points on the network may yield another shorter route than the existing one.

Hence the proof.

Mathematical equivalence: We will now establish that the above working requires no more than fourth degree polynomial time. We have to make two types of calculations:

- (i) $a + b - c$,
- (ii) $a + b - c - d$.

Now there are nC_2 segments which account for various 'c' subtractions also their are 'n' points for $a + b$. $\Rightarrow$ Maximum calculations could be $2n {}^nC_2$ as for one point and one segment we have two calculations. For 'n' points we have $2n$ calculations. Similarly for nC_2 segments we have in total $2n {}^nC_2$ calculations.

On similar grounds number of calculations for segment to segment are $2n {}^nC_2 \times {}^nC_2$

Order of polynomial P is

$$P = 2n \times {}^nC_2 + 2 \times ({}^nC_2)^2 \cong C \times N^4 + C \times N^3 \cong C \times N^4.$$

Hamiltonians path problem is solvable in polynomial time of fourth degree at most.